Digital Filters

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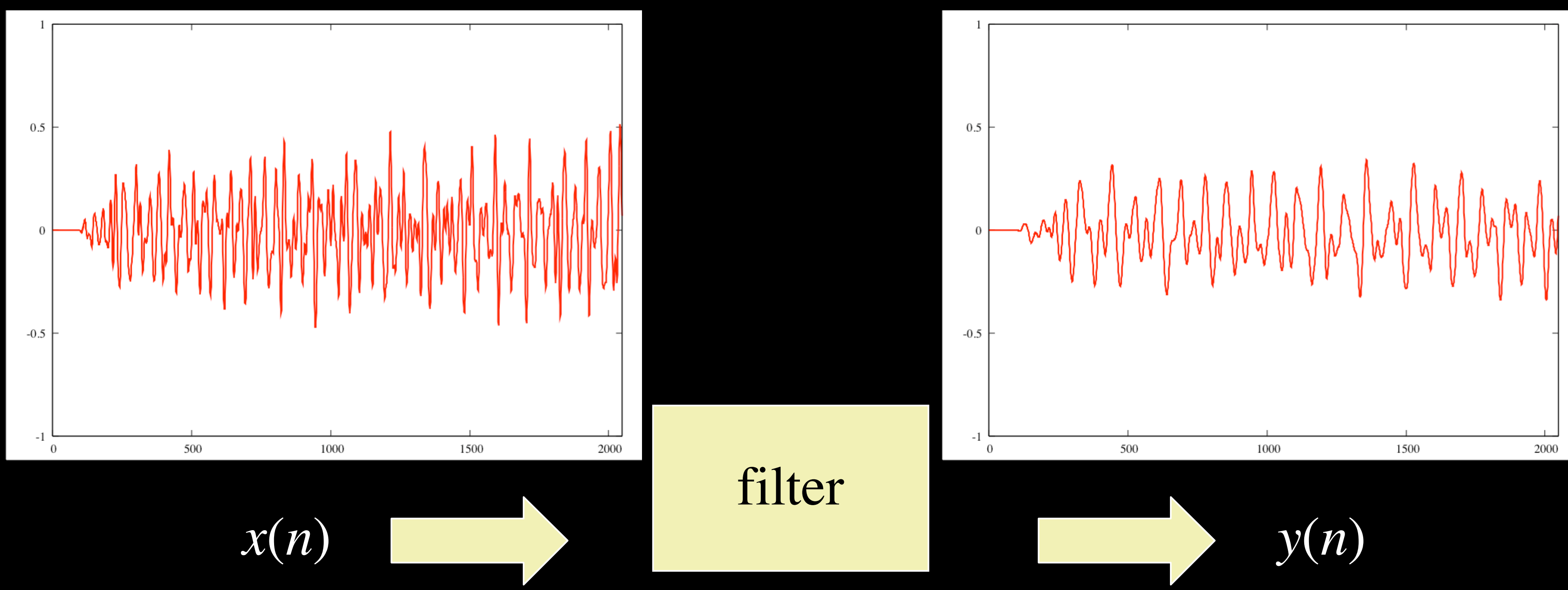
“black box” view of a digital filter

- audio signal represented as a mathematical function $x(n)$
  - maps sample number n to instantaneous amplitude

- input signal $x(n)$ process output $y(n)$

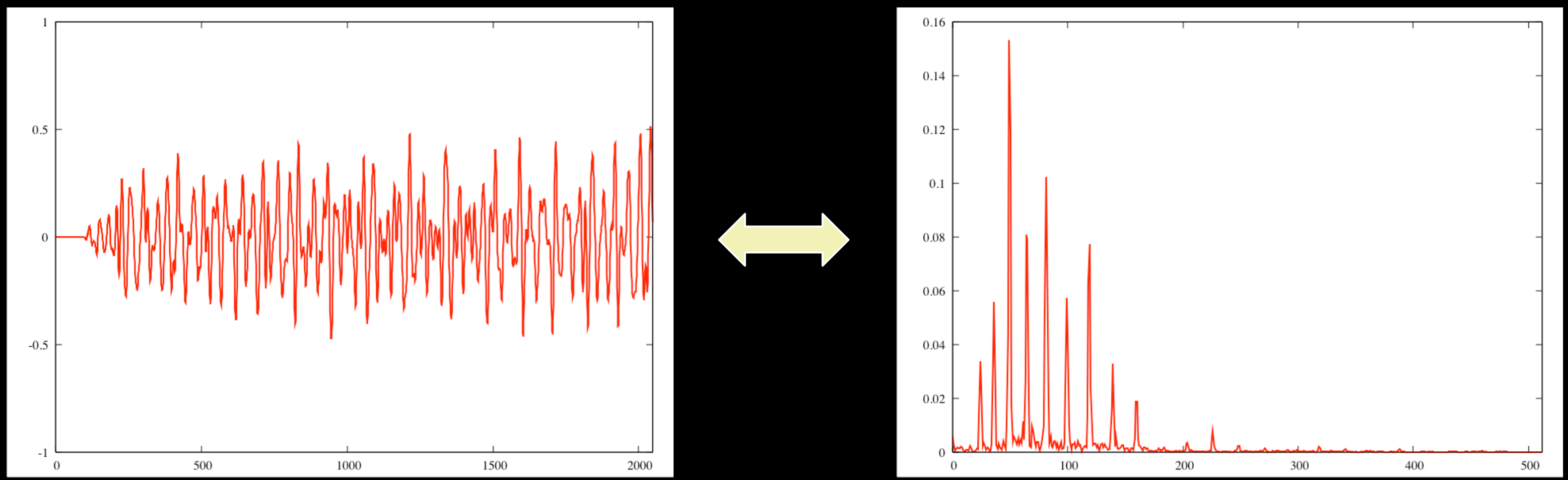


common applications

- modify the frequency spectrum of a signal
- apply frequency-dependent boost/cut
- emphasize or attenuate certain frequencies
- apply frequency-dependent phase changes

frequency spectrum

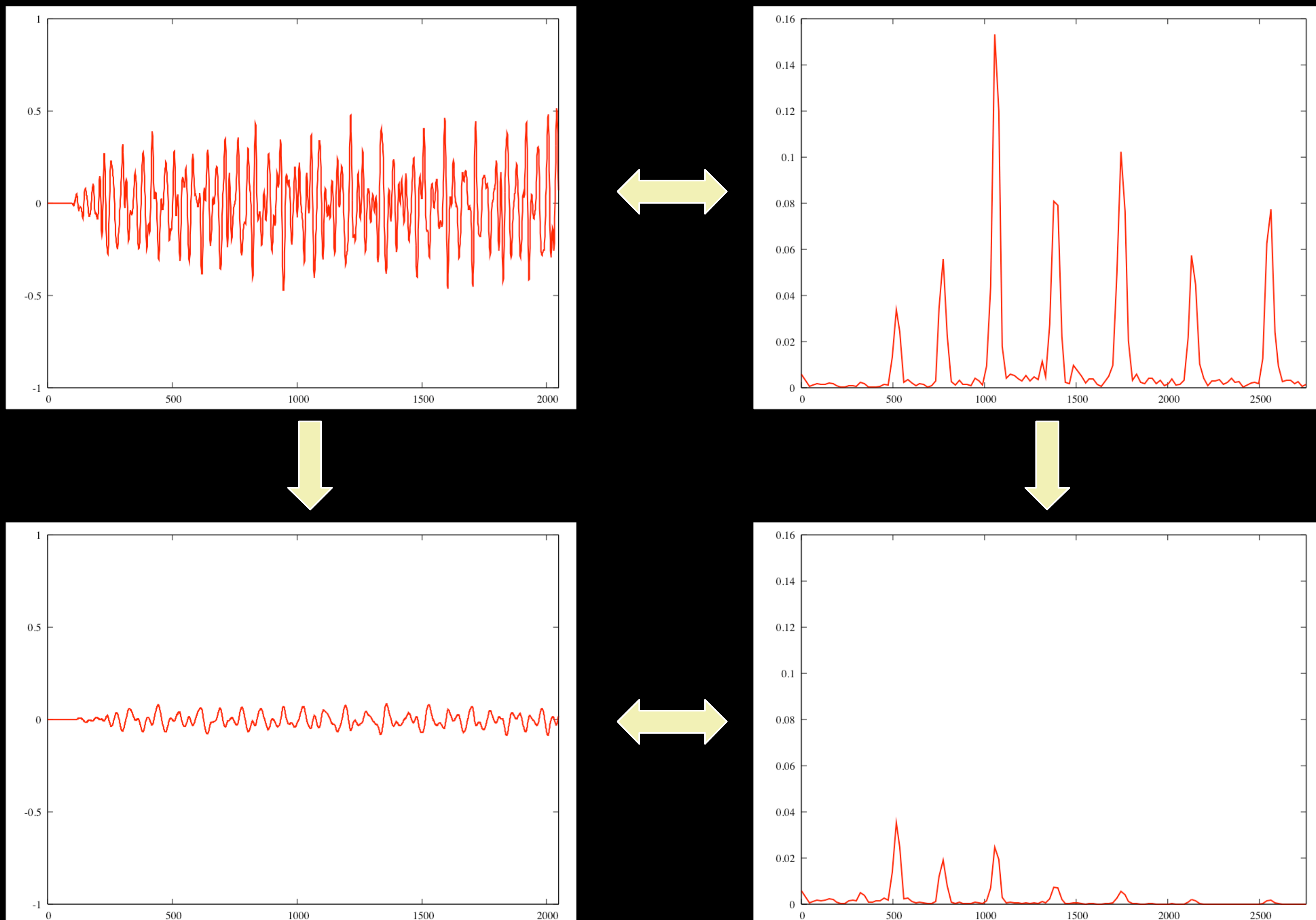
- representation of a signal in terms of sinusoids of specific amplitude, frequency and phase



- often graphed as frequency vs. amplitude (ignoring phase)
- the signal is no longer represented in terms of its evolution in time; it is represented in the *frequency domain*

example

- apply low pass filter (400 Hz cutoff, Q = 2.0)



sinusoids

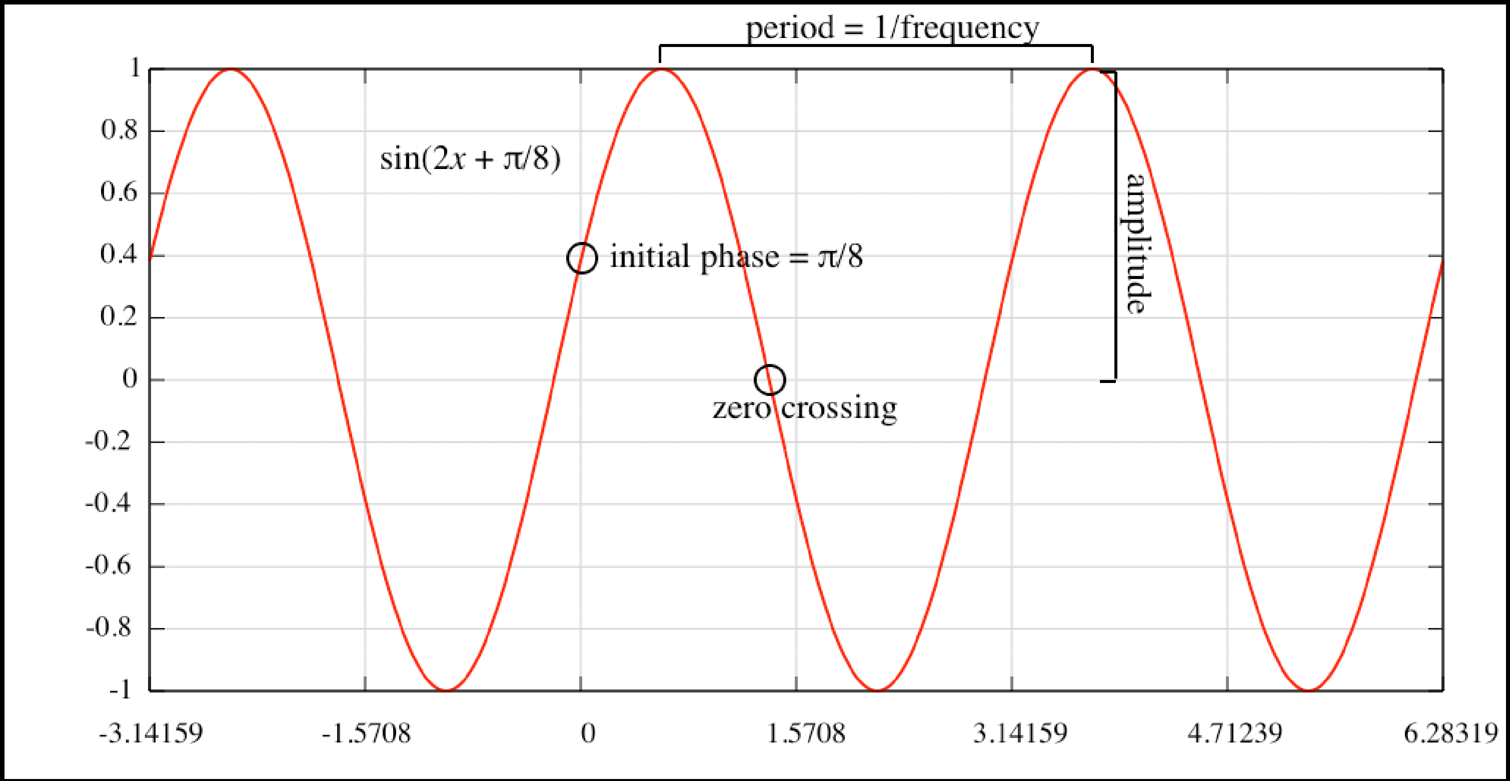
- $x(t) = A\sin(2\pi f t + \phi)$

  A = amplitude

  π = 3.14159… half the circumference of the unit circle

  f = frequency in cycles per second (Hz)

  ϕ = initial phase



significance of sinusoids

- objects that resonates or oscillate produces sinusoidal motion

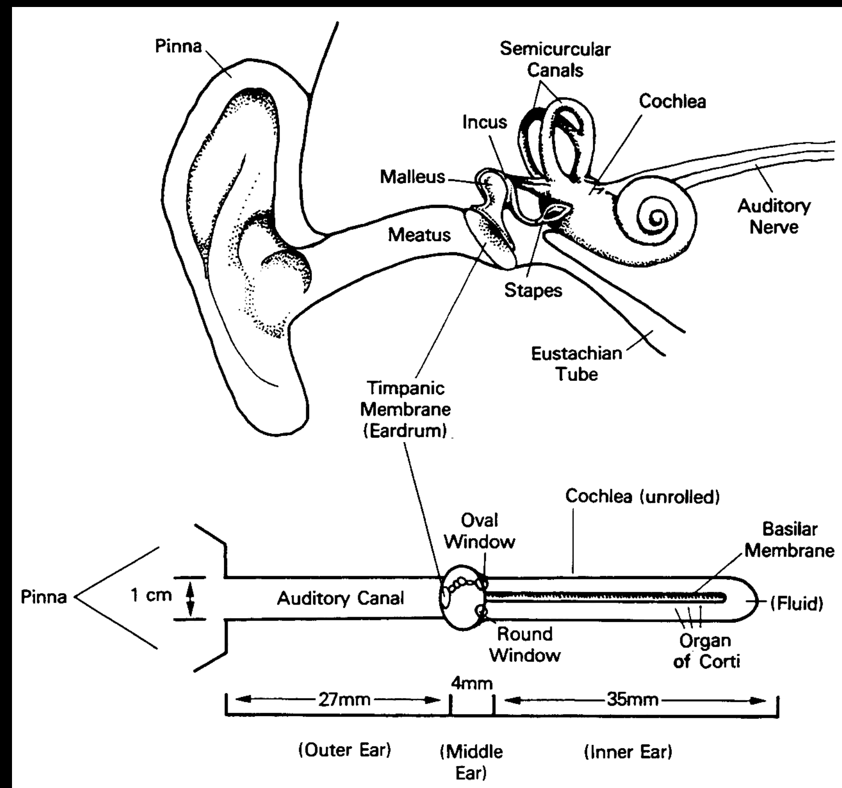
- simple harmonic motion
  - consider a point that vibrates back and forth on the x axis
  - acceleration is proportional to position
  - differential equation

$$\frac{d^2x}{dt^2} = -a^2x \quad \text{where } a \text{ is constant}$$

  - satisfied by $x = A\sin(at + \phi)$

the human ear as spectrum analyzer

- the cochlea of the inner ear separates sound into into (quasi) sinusoidal components
- vibrations produce compression waves in cochlear fluid
- basilar membrane
  - 30,000 hair cells
  - frequency specific nerve endings
- positions on the basilar membrane correspond to sinusoidal frequency



implementing digital filters

- most practical digital filters are implemented in the time domain
- they look at the current and past values of the signal to determine the output
  - FIR – Finite Impulse Response
- they may also look at the current and past outputs of the filter (feedback process)
  - IIR – Infinite Impulse Response

digital filter equation

- **for linear and causal digital filters**

$$y(n) = \sum_{i=0}^{M} a_i x(n-i) - \sum_{i=1}^{N} b_i y(n-i)$$

- $x(n)$ **is the filter input**
- $y(n)$ **is the filter output**
- a_i **and** b_i **the filter coefficients**

digital filter design problem

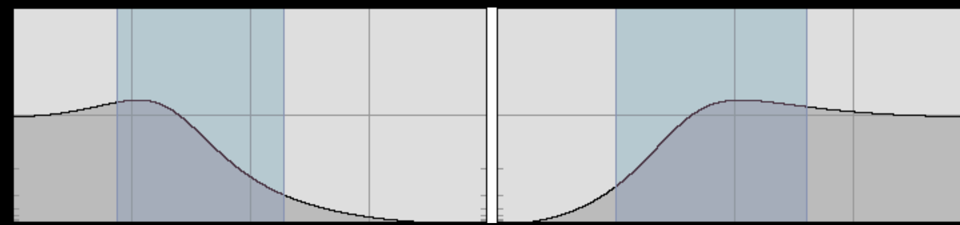
- how do we determine the filter coefficients (time domain) in order to achieve a particular frequency response?

- given a set of filter coefficients, what is the frequency and phase response of the filter?

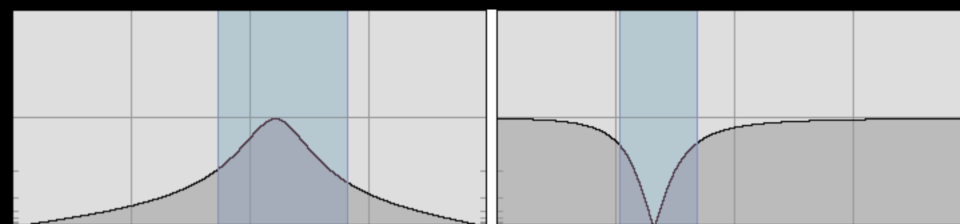
- use filter design algorithms

common filter types

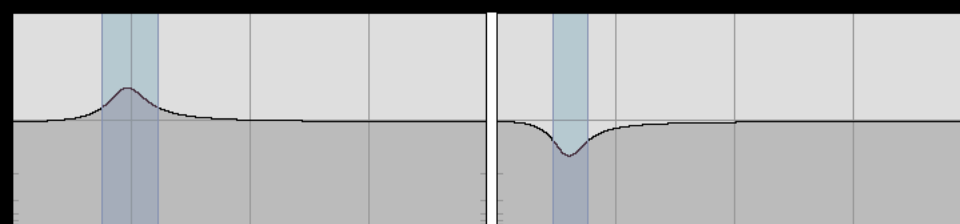
- lowpass/highpass



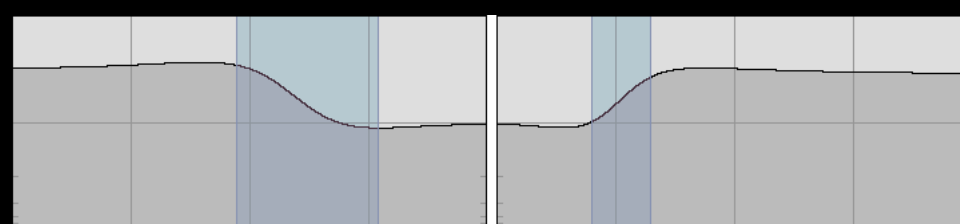
- bandpass/bandstop



- peak/notch



- lowshelf/highshelf



filter parameters

- frequency - in Hz
  - center frequency (bandpass, notch, resonators, etc.)
  - cutoff frequency (lowpass, highpass, shelving, etc.)

- bandwidth
  - width in Hz from the +/-3dB boost/cut point

- Q “quality factor”
  - center frequency divided by bandwidth
  - as Q increases the filter bandwidth decreases
  - resonance increases

filters in Max/MSP

- svf~, onepole~, reson~, ffb~, buffir~
- biquad~

$$
\begin{aligned}
y(n) &= \sum_{i=0}^{M} a_i x(n-i) - \sum_{i=1}^{N} b_i y(n-i) \\
&= a_0 x(n) + a_1 x(n-1) + a_2 x(n-2) - b_1 y(n-1) - b_2 y(n-2)
\end{aligned}
$$

- use filtergraph object to generate the 5 coefficients for biquad~

End

sine and cosine

- $\sin(\theta)$
  - **projection of the y component of a point on the unit circle rotated at angle** θ
- $\cos(\theta)$
  - **projection of the y component of a point on the unit circle rotated at angle** θ