Metaphor and Methodological Foundations of Atonal Theory

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Abstract

The principles of atonal pitch organization are still in large part unknown. The reason why they are unknown is also unknown. The present paper addresses this methodological problem. The paper first examines roles of metaphor in the theory of tonal accentual patterns and examines the way we handle music-theoretical issues with metaphor. Metaphorical expressions are used to classify and designate musical entities in music theory. In addition, they determine the course of the development of a theory. Since naïve set theory cannot by itself perform classification of musical entities, it seems likely that an atonal theory which designates and classifies currently unknown musical entities in atonal music comes with a collection of new metaphorical terms. It seems that David Butler’s “Intervallic Rivalry Model” and associated metaphorical expressions coined by Richmond Browne and Jonathan Kramer are useful for a theory of atonal pitch organization.

INTRODUCTION

The principles of atonal pitch organization are still in large part unknown.¹ The reason why they are unknown is also unknown. The present paper addresses this methodological problem and examines the nature of designation, classification, reasoning, and formalism in music theory, that is, the way we handle music-theoretical issues. It argues that metaphor plays crucial roles in the formation of a theory of atonal pitch organization and suggests a possible solution to the problem.

The problem is evident, for example, in the two different ways of segmentation employed by Ogdon (1981) and Forte (1981).

Example 1. Ogdon’s analysis, presented in his spelling

I use the term “segmentation” here to mean “the procedure of determining which musical units of a composition are to be regarded as analytical objects” (Forte 1973, 83). Ogdon’s segmentation of the opening passage from Schoenberg’s Drei Klavierstücke op.11/1 relies on principles of tonal pitch organization and he regards “chords” as “analytical objects.” Forte, on the other hand, considers as analytical objects those unordered pc-sets that are related to each other with respect to inclusion and complementary relationships.

Example 2. Forte’s analysis
These two analyses are among many conflicting attempts music theorists have made to identify “analytical objects” in atonal music, which I call “atonal kinds”: analytical or compositional units and their relations characteristic of atonal music. Issues of segmentation have been discussed in depth by Forte (1972, 1973, 1988) and Hasty (1981a, 1981b, 1984, 1986) and many theories of atonal pitch organization have tried to identify atonal kinds, which include the octatonic collection (Berger 1968, van den Toorn 1983, Cohn 1991, Forte 1994), reiterated pc-sets as a “tonic” (Travis 1959, 1966, 1970), extended “tonal kinds” designated by Schenkerian metaphors such as LINEAR PROGRESSION (Salzer 1962, Travis 1959, 1966, 1970), SYMMETRICALLY related pc-sets (Perle 1955, 1977, 1991, 1992, Antokoletz 1986), transpositionally related pc-sets (Cohn 1988, 1991b), and so on. Despite all of these invaluable attempts, however, we have not yet come to reach an agreement on what musical entities are atonal kinds and do not know how we can find them.4

The present study and perhaps most atonal theories as well presuppose that there exist some general principles of atonal pitch organization, that is, predetermined sets of musical entities and rules of their combinations and transformations. After so many attempts to find general principles, however, one might wonder if there are any consistent, general principles at all. There are two positions that would not accept the presupposition.

The first position is the contextualist view (Perle 1977, 162). According to this view, even if there exist some principles of atonal pitch organization, they are different from one piece to another and there are no general ones. If this is the case, however, there must exist some “principles of principles” of pitch organization, or the general principles, that enable us to find principles according to which pitch materials are organized within a particular piece and different sets of principles result in the diversity, or subclasses, of the atonal repertoire. We know, however, neither into what subclasses the repertoire should be classified nor how they should be identified and designated.

The second position finds no principles in atonal music. Lerdahl (1989, 84), for example, maintains that:

...atonal music is not very grammatical. ...Listeners to atonal music do not have at their disposal a consistent, psychologically relevant set of principles by which to organize pitches at the musical surface. As a result, they grab on to what they can: relative salience becomes structurally important. And within that framework the best linear connections are made.

There may be a class of atonal compositions to which we listen in this way. For example, consider the opening passage from the first movement of Webern’s Variationen für Klavier op.27, represented in different time for the clarity of the motivic formation:

Example 3. Webern, Variationen für Klavier op.27

The passage consists of four motives punctuated by sixteenth rests and, because of the superimposed retrograde series form, the “soprano” contours of the motives, or “salient linear connections,” are related to each other. That is, as Example 4 shows, the ascending and descending melodic figures of the first motive are coupled with the inverted ones of the second just like the opening theme from the first movement of Brahms’ Fourth Symphony and these coupled motives as a unit are further coupled with the next two so that they form a closure.5
If Lerdahl’s observation is correct, the very attempt to find the principles would be pointless and the assumption of their existence a misconception.

The entire repertoire of music labeled “atonal” seems, as suggested by the contextualist view, rather diverse, however. Consider the following example, comprised of Schoenberg’s spelling and elaboration of the same passage presented in Example 1:

Example 5. DIRECTED MOTION in Schoenberg’s op.11/1

Despite the awkward voice leading, we may perceive measure 3 to be like a half cadence in D minor, which is implied by Ogdon’s chord labeling in Example 1, and the passage to be DIRECTED towards the chord in m. 4, which is not present in Schoenberg’s score.6

The passage is not tonal, however. As Forte (1985) points out, Ogdon’s analysis does not make clear why the chord in m. 1 is the tonic in G major. Nevertheless, we rely on tonal analogies to classify and designate what we perceive in the atonal passage. For tonal music, only some particular pitch materials and relations are singled out from the entire range of sets of pitches and their relations and DIRECTED MOTIONS are created. It seems, therefore, that we have two related issues here. That is, we have no terms to designate and single out atonal kinds that cause DIRECTED MOTIONS in atonal music and do not know whether or not such MOTIONS result from intentional choices of particular pitch materials.

The choices of pc-sets and twelve-tone rows by some composers of atonal music are in fact intentional. For example, Table 1 shows the row classes Stravinsky chose for his twelve-tone compositions.
Since Stravinsky used the tone rows mostly as pairs of hexachords and did not strictly keep the order of the pitch classes in the hexachord, it seems possible to assume that he chose pairs of complementary unordered hexachords, as shown in Table 1, rather than twelve-note or pairs of six-note ordered pc-sets. Now, if only relative salience matters for atonal pitch organization, the choices of pc-sets and twelve-tone rows must be uniformly distributed. In other words, in the case of Stravinsky’s twelve-tone compositions, each Row Class must have the same relative frequency. So, let us examine the relative frequencies of all the twelve-tone series forms as pairs of $T_n/T_nI$ hexachords. Table 2 shows the relative frequencies of all the row classes:

<table>
<thead>
<tr>
<th>Row Class</th>
<th>Numbers of series forms</th>
<th>Relative frequencies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Row Class 1: 0 1 0 0</td>
<td>6!6! · 12 · 4 · 8 = 199,065,600</td>
<td>0.416</td>
</tr>
<tr>
<td>Row Class 2: 0 1 0 1</td>
<td>6!6! · 12 · 2 · 7 = 87,091,200</td>
<td>0.182</td>
</tr>
<tr>
<td>Row Class 3: 1 1 0 0</td>
<td>6!6! · 12 · 2 · 1 = 2,441,600</td>
<td>0.026</td>
</tr>
<tr>
<td>Row Class 4: 2 2 2 2</td>
<td>6!6! · 6 · 1 · 3 = 6,220,800</td>
<td>0.013</td>
</tr>
<tr>
<td>Row Class 5: 3 3 3 3</td>
<td>6!6! · 12 · 3 = 18,662,400</td>
<td>0.039</td>
</tr>
<tr>
<td>Row Class 6: 4 4 4 4</td>
<td>6!6! · 2 · 1 = 1,036,800</td>
<td>0.002</td>
</tr>
<tr>
<td>Total</td>
<td>12! = 479,001,600</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Table 2. Relative Frequencies of Row Classes

Now, Table 3 shows the relative frequencies of the tone rows Stravinsky chose:

<table>
<thead>
<tr>
<th>Row Class</th>
<th>Frequencies of tone rows</th>
<th>Relative frequencies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Row Class 1: 0 1 0 0</td>
<td>3</td>
<td>0.375</td>
</tr>
<tr>
<td>Row Class 2: 0 1 1 0</td>
<td>4</td>
<td>0.500</td>
</tr>
<tr>
<td>Row Class 3: 0 1 0 1</td>
<td>0</td>
<td>0.000</td>
</tr>
<tr>
<td>Row Class 4: 1 1 0 0</td>
<td>0</td>
<td>0.000</td>
</tr>
<tr>
<td>Row Class 5: 0 2 1 0</td>
<td>0</td>
<td>0.000</td>
</tr>
<tr>
<td>Row Class 6: 1 1 1 1</td>
<td>0</td>
<td>0.000</td>
</tr>
<tr>
<td>Row Class 7: 2 2 2 2</td>
<td>1</td>
<td>0.125</td>
</tr>
<tr>
<td>Row Class 8: 3 3 3 3</td>
<td>0</td>
<td>0.000</td>
</tr>
<tr>
<td>Row Class 9: 6 6 6 6</td>
<td>0</td>
<td>0.000</td>
</tr>
<tr>
<td>Total</td>
<td>8</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Table 3. Relative Frequencies of Stravinsky’s Row Classes
Statistically speaking, one would expect that, if Stravinsky chose his tone rows at random, their relative frequencies would be close to those in Table 2. The discrepancy between those two frequency distributions is significant, however. In fact, $x^2$ test shows that the probability of the choice of the tone rows by chance is approximately 0.5%. Thus Stravinsky’s choice is intentional. In other words, the particular pitch materials are deliberately chosen in Stravinsky’s twelve-tone music and the frequency distribution of the tone rows reflects his preferences for them.

Consequently, in some atonal music, there must exist principles—in this case, compositional constraints—that make composers choose certain pc-sets or tone rows over others. What is unknown here is, then, why and for what purpose they choose only those pitch materials. While music theorists have come up with principles of what Mead (1989) calls “taxonomies” such as those of set-classes and transformations, we still do not know those principles that govern composers’ choices of particular pitch materials or how atonal kinds can be identified and designated.

We designate tonal kinds, for example, with metaphorical terms such as SCALE, CHORD, RESOLUTION, MODULATION, and PROLONGATION. In fact, as Guck (1981) demonstrates in her pioneering work, metaphor permeates the music-theoretical discourse. In contrast to the other branches of music theory, however, metaphorical terms are rarely found in atonal theory. Therefore, in order to find why we know little about atonal kinds and how we can classify and designate them, it seems useful for us to examine how we identify and designate musical kinds with metaphors. In addition, since a theory is a product of a series of reasoning, it is essential for us to discuss the validity and soundness of reasoning with metaphors. For these reasons, my focus here is not on the cognitive process of the production of metaphor but on designation, classification, and reasoning with metaphors by which principles of atonal pitch organization are derived.

Consequently, although “An extensive body of recent work by cognitive scientists has suggested that . . . [conceptual models] provide essential guides to inference and reason” (Zbikowski 1997, 195), recent cognitive science is not very useful for the present study because it has little to do with forms of reasoning and their validity. For example, consider the following argument that uses the conceptual model “a chain-of-being hierarchy.”

If our reasoning is guided by a chain-of-being hierarchy, we infer that each musical work constitutes a domain pervaded by a mysterious force (Zbikowski 1997, 213).

The formulation of reasoning with metaphors is not so straightforward, however. This argument asserts that “we infer” from the premise “our reasoning is guided by a chain-of-being hierarchy” to the conclusion “each musical work constitutes a domain pervaded by a mysterious force.” Since “The fundamental conceit of chain-of-being hierarchies is to regard a domain as a huge organism pervaded by a [mysterious] force” (207), if a musical work is such a domain and the “force” pervading it remains of the same kind in the music domain, then “our inference” is a tautology, or a deduction, and hence the inference is necessarily valid and its soundness depends on the truth of the premise “we use a chain-of-being hierarchy.” In other words, as far as this particular inference is concerned, “essential guides” the conceptual model provides to the inference are its roles as a premise and the model has nothing to do with the form of the inference, or deduction. In this case, the recent work of cognitive scientists may be useful to support the premise and bolster the soundness of the inference. The soundness of the argument, however, depends on its premises “a musical work is such a domain” and “the force pervading it remains of the same kind in the music domain” as well as its form. Since a musical entity designated by a physical metaphor such as FORCE can be associated with different physical entities among music theorists, the “force” in the argument above may not remain of the same kind in the music domain. I shall later examine such conflicting usages of physical metaphors employed by Edward Cone and Jonathan Kramer. In addition to the problem of metaphorical association, an instantiation of “a domain” as a musical one in the argument is also problematic. This instantiation is based on a rule of inference called, in formal logic, “existential instantiation,” which cannot be used to derive a conclusion in a valid, or deductive, inference and yet is often found in music theory. In either of these cases, the argument above might no longer be valid or sound. It may involve a guess, an inductive generalization, or something else and we need to examine not conceptual models but the form of the reasoning so that we can evaluate its validity and soundness. I shall return to this point later when I examine Carol Krumhansl’s discussion of tonality.

Since the present study is primarily concerned with the way we derive principles of atonal pitch organization, the validity and the soundness as well as the forms of reasoning with metaphors by
which they are derived are its central focus. It is not in recent cognitive science but in the fields of formal logic, analytic philosophy, and philosophy of science that the validity and soundness of reasoning, issues of designation and classification, and the nature of theoretical discourse and its formal structure have most thoroughly been discussed. In this regard, the present study is partly in line and shares the same concern with Boretz 1970 that metaphor is an ambiguous, elusive device and might cause confusion. Unlike this monumental work, however, the present paper argues that metaphor is even indispensable for the formation of an atonal theory and it is crucial for us to explicate how we designate musical entities with metaphors and conduct sound reasoning with them.

Since no one knows what actually happens in our brain when we conduct classification, designation, and reasoning, we need to rely on some modeling. It is well-known that Aristotle examined reasoning patterns by analyzing the usage of our natural language. Formal logic, founded by him, is a “model” of reasoning and does not represent its actual process in our brain. “The reason logic is relevant to knowledge representation and reasoning is simply that . . . logic is the study of entailment relations—languages, truth conditions, and rules of inference” (Brachman and Levesque 2004, 11). For these reasons, I shall restrict myself to naïve set theory and associated formal logic to represent metaphors.\textsuperscript{15}

Although Lakoff and Johnson (1980, 122) and Lakoff (1987) maintain that set theory is inappropriate to represent metaphors, some forms of reasoning with metaphors can be represented, as shown later, only by extensional means, that is, in set-theoretical terms. While Johnson (1997–98), Larson (1997–98, 2002), Mead (1997–98, 1999), Saslaw (1996, 1997–98), and Zbikowski (1997), to name a few, discuss metaphor in music theory from the viewpoint of Lakoff and Johnson 1980 and Johnson 1987, studies of the forms of reasoning with metaphors are rarely found in music theory. From an extensional, or set-theoretical, point of view, classification, designation, and reasoning are interrelated. Since reasoning with metaphor plays crucial roles in theory formation, set-theoretical representation of metaphor is indispensable for the present study.

Another reason for the use of naïve set theory in the present paper is to discuss what it can and cannot do in music theory. It is the very limitation of the set-theoretical representation that reveals not only what roles metaphor plays in music theory but also an implicit methodological assumption pc-set theory observes. In short, naïve set theory will be used for two purposes: first, to explain how metaphor works; and, second, to show its roles in music theory.

Since few metaphorical expressions are currently available in atonal theory, to explicate their roles in designation, classification, reasoning, and theory formation, we have to start examining metaphors in a field of research where they are already employed.

**PHYSICAL METAPHOR AND THE BALANCE PRINCIPLE**

Metaphorical expressions in music theory result from associations between musical entities or experiences and extra-musical referents.\textsuperscript{16} Such associations are, however, by no means inherent in musical entities themselves but arbitrary to some extent and may vary from one speech community to another. For example, A NEIGHBOR NOTE is called “une broderie,” or AN EMBELLISHING NOTE, in French, a 40dB sound is SOFT in English while it is SMALL in Japanese, and so on.

Even within a single speech community, for instance, that of English-speaking music theorists, metaphorical associations might vary from one speaker to another. Especially in a new or rapidly growing research field such as that of rhythm and meter, for which new metaphorical terms are coined, it is not uncommon for the referent of a single metaphorical expression to be associated with different musical entities among different music theorists. Much discussed disagreements on accentual patterns of the opening passage from Mozart’s *Sonata* K. 331, for instance, may originate in such different metaphorical associations.\textsuperscript{17} In order to examine how metaphor works in music theory, it is useful to recall Kramer’s discussion (Kramer 1988) about accentual patterns in a typical four-bar phrase because the conflicting beliefs among theorists are expressed in metaphorical terms and the controversy may be familiar to readers.

Kramer (1988, 84ff) begins his discussion with the classification of accentual patterns as follows:
Example 6. Three Accentual Patterns in a Four-Bar Phrase

He believes that disagreements over these accentual patterns stem from different understandings of the notion of accent. To solve the disagreements, therefore, Kramer defines, in part following Lerdahl and Jackendoff (1983), three different types of accent as follows: A “stress accent” is “the emphasis on a note created by a sharp attack, a high dynamic level, a small preceding silence, and so forth.” A “rhythmic accent” is “a point of stability . . . (one of) the focus(es) of a rhythmic group, . . . a cadence is typically . . . a point of rhythmic accent.” A “metric accent” is “a point of initiation [of a metric timespan, or (hyper)measure]” (Kramer 1988, 86). While a “stress accent” can be identified by its physical property, rhythmic and metric accents have nothing to do with physical force and yet are associated with something physically strong or weak. The expressions A STRONG ACCENT and A WEAK ACCENT are, therefore, metaphors designating some musical entities.

With the notions of these three types of accent, Kramer (1988, 88ff) examines Cone’s analysis of the opening phrases from Mozart’s Sonata K.331. He argues that Cone (1968) identifies A STRONG ACCENT in the first rhythmic group in m. 5 because of the STRONG HYPER-METRIC ACCENT of the two-bar unit of mm. 5–6 but regards the last rhythmic group also as STRONG because of its rhythmic accent. Moreover, while Cone describes the third rhythmic group in m. 7 as WEAK, Kramer feels that there is A STRONG HYPER-METRIC ACCENT in the same measure. Kramer maintains that Cone does not clearly state whether accents are applied to timepoints or timespans and thereby fails to distinguish a hypermetric accent from a rhythmic one. He concludes that the first accentual pattern supported by Cone results from confusion between the two different types of accent.

Kramer’s metrically and rhythmically STRONG and WEAK ACCENTS may be characterized as what Carbonell and Minton (1985) call “physical metaphor.” They argue that physical metaphors conduct “reasoning about imponderable or abstract entities as though they were objects...” (407). Consequently, “Inference patterns valid in physical domains are transformed into inference patterns applicable in different target domains . . . preserving underlying relations such as causality” (408). Many metaphors found in music theory are of this type. In the metaphor A TONIC IS STABLE, for example, a tonic bears a property normally attributed to something physically stable.

Reasoning with physical metaphors is carried out by way of transferring some prominent properties, relations, or principles in the physical domain to the music domain and assuming that those properties or principles are operative in the music domain. Carbonell and Minton (1985, 407–8) observe that one of those properties likely to be transferred from the physical domain is what they call “the balance principle.” The principle maintains that, when we have some metaphorical PHYSICAL OBJECTS that exert metaphorical PHYSICAL FORCE in a certain closed system, the state of the system is conditioned in such a way that the PHYSICAL FORCE will eventually reach EQUILIBRIUM. According to Cone (1968, 26–27) calls MUSICAL ENERGY, in a four-bar phrase. The COUNTERFORCE should be designated by the antonym of STRONG, namely, WEAK.

Consequently STRONGs and WEAKs have to be of the same type so that they can be counterbalanced by each other. By contrast, if the balance principle is not transferred, accents do not have to keep BALANCE. Hence, in this case, whether the differences among the types of accents defined by Kramer are taken into account or not, they do not have to be of the same type and can be timepoints or timespans. Finally, if one does not accept the differences among the types of accents, it follows that the accents are of a single type and therefore “confusion” over different types cannot happen whether the transfer of the principle takes place or not.
Since Cone (1968, 26–27) refers to “an initial downbeat” and “a cadential downbeat” as “strong points,” he also seems to acknowledge differences among the types of accents. Therefore, if the balance principle were transferred in Cone’s STRONGs and WEAKs, he would be—as Kramer argues—confused about the distinction between hypermetric and rhythmic accents. It seems more likely, however, that, while the principle is transferred in Kramer’s metaphors, it is not in Cone’s. In this case, it would not matter to Cone if STRONGs are not counterbalanced by WEAKs of “the same type.” Thus, if Cone used instead of STRONG such terms as “salient” and “prominent,” the disagreement between Kramer and Cone would not occur since these terms do not have their antonyms as their physical counterparts and hence are not subject to the balance principle. So, in the end, the issue of the accentual patterns discussed by Kramer and Cone seems relevant to the metaphoric transfer of the balance principle. To discuss whether the transfer of the principle is justified or not, it is necessary for us to examine the internal working of metaphor.

CLASSIFICATION AND DESIGNATION WITH METAPHOR

Several theories of metaphor such as comparison theory, substitution theory, tension theory, controversy theory, and deviance theory have been proposed since Aristotle’s Poetics. Until a couple of decades ago, however, metaphor had been regarded only as a marginal, imaginary, and thus inaccurate linguistic device and not as a subject of serious inquiry in philosophy. This situation has changed since Max Black’s interaction theory, which was initially proposed by Richards (1936a, 1936b) and gave the first important philosophical insights into the roles of metaphor.

Black’s interaction theory is still most influential in philosophy, psychology, and computer science and discussed by many researchers such as Way (1991), Glicksohn and Goodblatt (1993), Sokice and Harré (1995), Steinhart (2001), and Indurkhya (1992, 1994, 2006). His own explanation of the theory, however, often relies on metaphor, which some philosophers criticize. Thus, some formal language is necessary for an application of interaction theory and representing metaphors so that those processes are independent of particular metaphorical expressions in a natural language.

Using the metaphor MAN IS A WOLF as an example, Black (1962a, 40) explains his interaction theory as follows:

Literal uses of the word normally commit the speaker to acceptance of a set of standard beliefs about wolves . . . that are the common possession of the members of some speech community. . . . The idea of a wolf is part of a system of ideas. . . .

It should be emphasized that only within a particular speech community, can a set of standard beliefs be, to a certain extent, determined. Each belief about the referent of a word may assume that the referent has some properties.

Now, let $M = \{m_1, m_2, \ldots \}$ and $W = \{w_1, w_2, \ldots \}$ be sets of predicates denoting properties which the members of a given speech community believe are possessed by “man” and “wolf” respectively. Then, $Man = \{x | m_1(x) \land m_2(x) \land \cdots \}$ and $Wolf = \{x | w_1(x) \land w_2(x) \land \cdots \}$ are the classes $Man$ and $Wolf$. The standard beliefs about $Man$ and $Wolf$ are accordingly represented by the sets of their most prominent properties $s_M = \{m_1, m_2, \ldots, m_j \}$ and $s_W = \{w_1, w_2, \ldots, w_g \}$. That is to say, there is a partial order between a pair of members in $M$ and in $W$ with respect to prominence such that $m_{i+1} \leq m_i$ for $1 \leq i < j$. That is, $m_i$ is more prominent than or as prominent as $m_{i+1}$. The beliefs about $Man$ and $Wolf$ are, therefore, represented as ordered sets $(M, \leq)$ and $(W, \leq)$. Since more prominent properties represent “associated standard beliefs,” “standard” man, $s_{Man}$, and wolf, $s_{Wolf}$, should be as follows:

\[
s_{Man} = \{x | m_1(x) \land m_2(x) \land \cdots \land m_j(x) \}
\]

\[
s_{Wolf} = \{x | w_1(x) \land w_2(x) \land \cdots \land w_g(x) \}.
\]

Here, for instance, $w_1(x)$ could be “$x$ is fierce”; $w_2(x)$, “$x$ is carnivorous”; $w_3(x)$, “$x$ has fangs”; and so on.

Carbonell and Minton (1985, 407) postulate the components of metaphor and their functions as follows: “A metaphor, simile, or analogy can be said to consist of 3 parts: a target, a source
and an analogical mapping.” For example, in the simile “John was embarrassed. His face looked like a beet,” the target is “John’s face” and the source is “a beet.” An analogical mapping transmits information from the source to the target domain. That is, here some information about a beet is mapped into the target domain.

Two domains also come into play even when a metaphor does not take the form “S(subject) is P(redicate)” or, more generally, “S [verb] P.” For example, in the metaphor AN ASCENDING PASSAGE, a musical event is associated with a physical object ascending in space, and so, in this case, the target is a “passage” and the source is an object ascending in space.

A mapping, or a metaphoric transfer, is triggered because, as Davidson (1984b, 258) points out, when a sentence is taken to be false or contradictory, we start to hunt out the hidden implication. Because of our belief:

\[ \{x \mid s \text{Man}(x)\} \cap \{x \mid s \text{Wolf}(x)\} = \emptyset \]

the sentence “Man is a wolf” is literally contradictory. Then, a search for “the hidden implication” sets off. When we have a contradiction of this sort, we attempt to find properties \( S = \{s_1, s_2, \ldots, s_n\} \) such that \( S = M \cap s \text{Wolf} \). Needless to say, \( \{m_1, m_2, \ldots\} \cap \{w_1, w_2, \ldots\} \neq \emptyset \), that is, Man and a wolf are both warm-blooded, social, and so on. In other words, some prominent properties in the target domain that correspond to standard beliefs about Man become suppressed while some of those that do not correspond, i.e., the set \( S \) are highlighted. As a result, the order in \( (M, \leq) \) changes according to that in \( (s \text{Wolf}, \leq) \). Some of the less prominent properties in the target, which are prominent in the source, become highlighted as if they were transferred from the source domain to the target domain. In short, the greater the degree of prominence of a property in the source domain, the more likely the property is transferred to the target domain. Here, for example, \( s_1 \) and \( w_1 \) might be “being savage.”

After the reorganization of the order in \( (M, \leq) \), a new set \( m \text{Man} \), or metaphorical MAN, such that:

\[ m \text{Man} = \{x \mid s_1(x) \land s_2(x) \land \cdots \land m_1(x) \land m_2(x) \cdots\} \]

and hence \( m \text{Man} \subset \text{Man} \) is obtained. In other words, the reorganization in the target domain caused by a metaphoric transfer results in a subclassification of the class designated by the target word. A metaphor creates a new subclass of the referent of the target, which is in turn metaphorically designated by the same target word. Black (1962b, 236) explains this process of reorganization as follows:

The effect . . . of (metaphorically) calling a man a ‘wolf’ is to evoke the wolf-system of related commonplaces. . . . A suitable hearer will be led by the wolf-system of implications to construct a corresponding system of implications about the principal subject [target]. . . . The wolf-metaphor suppresses some details, emphasizes others—in short, organizes our view of man.

It follows that one of the crucial roles metaphor plays in music theory, designating musical entities, is accomplished through this subclassification. By way of a metaphoric transfer, it is a new class of musical entities that the target word designates in a metaphor. So, the fact that we use a number of metaphorical expressions in music theory implies that we have performed the same number of new classifications so that we can designate previously unknown musical entities.

**DEDUCTION WITH METAPHOR**

Since metaphors create subclasses, hence inclusion relationships among the classes designated by target words, there are necessarily deductive implication relationships among metaphors. An example of such relations in Kramer’s argument can be shown in the following way: Since an accent is either STRONG or WEAK, he must implicitly presuppose the metaphor ACCENTS EXERT FORCE. Then, in order to justify this metaphor, he needs to presuppose another metaphor ACCENTS ARE PHYSICAL OBJECTS, because “force” is in its literal sense exerted by physical objects. This second metaphor will be in turn justified if the general metaphor MUSICAL ENTITIES ARE PHYSICAL OBJECTS is assumed. This metaphor is “general” in that it is most abstracted and hence inclusive. Consequently, since A MUSICAL ENTITY IS A PHYSICAL OBJECT, which exerts physical force, and an accent is a musical entity, AN ACCENT ALSO EXERTS FORCE, and hence it is STRONG or WEAK. Consequently, Kramer’s theory
of rhythm and meter can be conceived as a deductive system of metaphorical expressions, which represents his beliefs and is derived from the single, general metaphor MUSICAL ENTITIES ARE PHYSICAL OBJECTS.

Needless to say, it is unlikely that Kramer first conceived this general metaphor himself and then deductively derived the other metaphors one by one. The point here is that, from a formal point of view, if the general metaphor is true in some sense, the other physical metaphors deductively derived from it will be necessarily justified, and his entire theory will form a coherent, deductive system. So, let us examine next how the general metaphor contributes to the consistency of Kramer’s theory.

It is possible, as sketched out above, to reformulate Kramer’s theory in such a way that his belief METRIC ACCENTS ARE SUBJECT TO THE BALANCE PRINCIPLE is derived from the following reasoning:

\[
\begin{align*}
\text{MUSICAL ENTITIES ARE PHYSICAL OBJECTS.} \\
\text{Metric accents are subject to the balance principle.}
\end{align*}
\]

The process of this reasoning can be represented as a series of deductive inferences using other related metaphors and nonmetaphors:

\[
\begin{align*}
\text{MUSICAL ENTITIES ARE PHYSICAL OBJECTS.} \\
(1) & \quad \text{Metric entities are musical entities.} \\
& \quad \text{METRIC ENTITIES ARE PHYSICAL OBJECTS.} \\
\text{METRIC ENTITIES ARE PHYSICAL OBJECTS.} \\
(2) & \quad \text{Metric accents are metric entities.} \\
& \quad \text{METRIC ACCENTS ARE PHYSICAL OBJECTS.} \\
\text{Physical objects are subject to the balance principle.} \\
(3) & \quad \text{METRIC ACCENTS ARE PHYSICAL OBJECTS.} \\
& \quad \text{Metric accents are subject to the balance principle.}
\end{align*}
\]

Example 7. Derivation of Kramer’s Belief about Metric Accents

These inferences rely on the transitivity of class-inclusion relationships among the extensions of the terms. The reasoning pattern of the inference (1), for example, can be represented in terms of class-inclusion relationships as follows:

\[
\begin{align*}
\text{Musical Entity} & \subset \text{Physical Object} \\
\text{Metric Entity} & \subset \text{Musical Entity} \\
\text{Metric Entity} & \subset \text{Physical Object},
\end{align*}
\]

where

- \text{Metric Entity:} the class of metric entities
- \text{Physical Object:} the class of physical objects
- \text{Musical Entity:} the class of musical entities.

It should be noticed that, when this inference is carried out, as discussed earlier, a new subclass of \text{Musical Entity} is created by the general metaphor MUSICAL ENTITIES ARE PHYSICAL OBJECTS. That is to say, if we have the classes of musical entities and “standard” physical objects such that

\[
\begin{align*}
\text{Musical Entity} & = \{x|u_1(x) \land u_2(x) \land \cdots\} \\
\text{s Physical Object} & = \{x|p_1(x) \land p_2(x) \land \cdots \land p_9(x)\},
\end{align*}
\]

then, through metaphoric transfer, metaphorical \text{Musical Entity:}

\[
\begin{align*}
m_{\text{Musical Entity}} & = \{x|p_1(x) \land p_2(x) \land \cdots \land u_1(x) \land u_2(x) \land \cdots\}
\end{align*}
\]
is created, and hence $m_{Musical\text{-}Entity} \subset Musical\text{-}Entity$.

Furthermore, because of the relation $Metric\text{-}Entity \subset Musical\text{-}Entity$ and the consequence of the inference (1), when

$$Metric\text{-}Entity = \{x | r_1(x) \land r_2(x) \land \cdots \land u_1(x) \land u_2(x) \cdots\},$$

metaphorical $Metric\text{-}Entity$ such that:

$$m_{Metric\text{-}Entity} = \{x | p_1(x) \land p_2(x) \land \cdots \land r_1(x) \land r_2(x) \land \cdots \land u_1(x) \land u_2(x) \cdots\}$$

is created. Likewise, because of the consequence of the inference (2), the class of STRONG or WEAK metric accents:

$$Metric\text{-}Accent = \{x | a_1(x) \land \cdots \land r_1(x) \land \cdots \land u_1(x) \land \cdots\}$$

is included as

$$m_{Metric\text{-}Accent} = \{x | p_1(x) \land \cdots \land a_1(x) \land \cdots \land r_1(x) \land \cdots \land u_1(x) \land \cdots\}$$

in its superclass $m_{Metric\text{-}Entity}$. Consequently, there are class-inclusion relationships among Kramer’s metaphors, which are represented as follows:

$$m_{Metric\text{-}Accent} \subset m_{Metric\text{-}Entity} \subset m_{Musical\text{-}Entity},$$

and transferred properties are inherited from the superset to its subsets:

$$m_{Musical\text{-}Entity} = \{x | p_1(x) \land \cdots \land u_1(x) \land \cdots\}$$

$$m_{Metric\text{-}Entity} = \{x | p_1(x) \land \cdots \land r_1(x) \land \cdots \land u_1(x) \land \cdots\}$$

$$m_{Metric\text{-}Accent} = \{x | p_1(x) \land \cdots \land a_1(x) \land \cdots \land r_1(x) \land \cdots \land u_1(x) \land \cdots\}$$

In short, there is a class-hierarchy among classes designated by the music-theoretical terms, and the subclassification of a class carried out in a general metaphor and its related metaphoric transfer affect all subclasses of the class as if “metaphoness” in a superclass were inherited in its subclasses.

This inheritance of metaphoric transfer in the class-hierarchy also enables the deductive consistency of Kramer’s argument about hypermetric regularity (Kramer 1988, 100) as shown in Example 8:

**Example 8. Derivation of Kramer’s Belief about Hypermeter**

Note that this series of inferences shares the inference (1) in Example 7 because both the class of hypermeter and $Metric\text{-}Accent$ are subsets of $Metric\text{-}Entity$. Perhaps implicitly guided by the metaphoric transfer of the balance principle to the domain of hypermeter, which is supported by this series of deductions, Kramer (1988, 98–102) has acquired a new belief that metric irregularity, or PHYSICAL IMBALANCE, may be resolved on a higher metric level, i.e., WEAK
and STRONG HYPER-METRIC ACCENTS counterbalance each other. This is an instance of knowledge acquisition by deduction. In this respect, too, Kramer’s reliance on the balance principle, which has caused the disagreement with Cone, is evident.

The series of deductive inferences based on the class-hierarchy, or the inheritance of metaphoric transfer, is the underlying formal, namely, in this case, deductive, structure of Kramer’s theory of rhythm and meter, by which the theory is given consistency. When a theory is deductively consistent, the soundness of the theory depends on whether the general metaphor, that is, the metaphor carrying a term that is the root of a class-hierarchy, is true in some sense. In other words, the soundness of a deductively consistent theory has to do with metaphoric transfer in a general metaphor. Still unclear is, therefore, how an instance of metaphorical transfer is justified. Also puzzling is why we have difficulty in acquiring new beliefs about atonal kinds using pc-set theory.

ROLES OF MATHEMATICS IN MUSIC THEORY

The production of a metaphorical term in music theory is, as we have observed, necessarily concurrent with subclassification of musical entities. As a result, some metaphorical terms used in current tonal theories, for example, might suppress properties particularly characteristic of atonal music. If this is the case, one way to designate those suppressed properties is to generalize through abstraction, or class-inclusion relationships, those terms so that they designate classes of any musical entities. Pc-set theory has developed exactly in this way. The extensive use of symbols representing more inclusive classes of musical entities is a natural consequence of the development of the theory. As a result, pc-set theory provides us with powerful tools to represent pitch materials and their relations. The formalism of pc-set theory, in this case, an application of naïve set theory to music theory, has, however, serious side effects, which are also evident in the set-theoretical representation of metaphor.

The way of representing metaphors employed in the previous sections relies on the following assumption: Natural kinds such as man and the wolf can be represented as sets of particular entities. Any entities can be brought together into a set, however. Consider, for example, the following three sets: a set of five apples; a set of four organs and one chocolate parfait; a set of Palestrina, Bach, Ravel, Stravinsky, and Kern. Since all three sets share the common property of having five elements, they are members of the same superset. Needless to say, since a natural kind is not an arbitrary set like this, there must be some conditions or criteria that all the members of a set must satisfy so that the set is a kind. Such conditions must be the properties that all the members of a set have in common. Consequently natural kinds could be represented by listing each and only those properties, or intensions, that all the members of a set share.

This assumption is, however, susceptible to several serious flaws. In the first place, although set Man can be defined with respect to its properties, they do not necessarily represent our standard beliefs about man. Man can be sufficiently defined in any of, but not limited to, the following ways:

\[\text{Man}_1 = \{x|\text{Walking Upright}(x) \land \text{Being a Mammal}(x)\}\]
\[\text{Man}_2 = \{x|\text{Using a Doubly-Articulated Language}(x)\}\]
\[\text{Man}_3 = \{x|\text{Having a Human DNA}(x)\}\]

Obviously, not only \(\text{Man}_2\) and \(\text{Man}_3\) but also even \(\text{Man}_1\) would not typically represent our standard beliefs about man. That is, just specifying sufficient properties is inadequate to represent \(s,\text{Man}\). Therefore, there must be a set of some other properties that play a major role in determining the class \(s,\text{Man}\). So, our task might be to find a set of appropriate properties that do not conflict with our standard beliefs.

It is, however, even impossible to use properties as criteria that distinguish kinds from arbitrary sets, for properties themselves can also be put together in any arbitrary way. For example, the following is a set of creatures defined by two properties:

\[\{x|\text{Using a Doubly-Articulated Language}(x) \land \text{Having Four Feet}(x)\}\]

These strange creatures share the same properties with human beings and four-foot animals. So, if kinds were defined solely by properties, we would find the creatures similar to either of
them. Needless to say, however, we would neither believe that the creatures and human beings are similar to each other nor believe that they are of the same kind. We believe that the set of the creatures is not a kind simply because we already know that those two properties cannot be put together as those with respect to which a kind is defined.33

Note that arguments like the foregoing are common in music theory. It is common practice in pc-set theory and transformational theory to define similarities with respect to shared properties such as the number of common pitch classes or interval classes and to classify transformations, set-classes, or their relations accordingly because it is assumed that those “similar” entities or relations should belong to the same set and form a musical kind. Musical entities thus classified might be musical counterparts to the strange creatures, however. Even if two different pc-sets or transformations share the same properties, they are not necessarily similar to each other for the same reason that the strange creatures and human beings are not similar to each other.34 Likewise, those pitch classes or pc-sets that may be considered close in a certain representation of PITCH SPACE might not be CLOSE to each other in terms of aural perception.

Although, in principle, we can conceive an infinite number of any sets, as shown earlier, by specifying any combinations of any properties—so-called “ontological inflation”; obviously not all sets so conceived can be kinds. In order to avoid such devastating ontological inflation, we usually impose some order on entities and conduct classification. Prior to classification, however, we need to know what is the order and what properties are the determining factors for classification. In short, it is in this sense trivial to say that kinds and standard beliefs associated with them are defined and represented by properties.35

The notions of similarity, kind, and category are, as Quine argues, “substantially one notion” and “alien to logic and set theory” (1969, 117–21). The set-theoretical representation itself reveals that it cannot make clear how kinds are formed, nor can it represent their criteria. In short, “Similarity cannot be equated with, or measured in terms of, possession of common characteristics” (Goodman 1972, 443). “The grouping of occurrences under a work or an experiment or an activity depends not upon a high degree of similarity but upon the possession of certain characteristics” (440; emphasis added).36 Naïve set theory is not a method we use to identify atonal kinds and to produce and understand metaphors.

On the other hand, set-theoretical representation is even indispensable for the analysis of some forms of reasoning with metaphors and acquiring new beliefs. Lakoff and Johnson (1980) schematize what they call a typical metaphorical reasoning pattern as follows:

\[
\begin{align*}
F(A) \\
A = B \\
F(B)
\end{align*}
\]

that is, “A is F” and “A equals B,” therefore “B is F,” where “A equals B” is a metaphor. Note that, in this representation of a deductive inference, extensional relationship is hidden. In classical logic, the type of sentence “S is P” was regarded as the basic form of sentence, which consists only of what Bertrand Russell calls single-term propositional function, or the type of \( F(x) \), where F is a predicate, and is used in syllogism. Problems concerning the representation of metaphor happen, therefore, when a metaphor consisting of multi-term relations \( F(x_1, x_2, \ldots, x_n) \) is analyzed. Lakoff (1993, 212–13) refers to another reasoning pattern with metaphor:

A is in B.
X is in A.

\[
\begin{align*}
X & \text{ is in } B.
\end{align*}
\]

This is also one of the modes of syllogism known as the Darii mode. Here, what is transferred, that is, how a subclass is created, is not clear.

In addition to these two modes of syllogistic inferences just mentioned, there are many other valid reasoning patterns with metaphors that cannot be handled by classical logic. For instance, classical syllogism cannot handle the following inference, which shows the inheritance of the balance principle discussed earlier:37
A METRIC ACCENT IS A PHYSICAL OBJECT.

A PROPERTY OF A METRIC ACCENT IS THAT OF A PHYSICAL OBJECT.

There are many other inference patterns like this that cannot be represented without the notions of set and class. This inference is extensionally formulated as follows:

\[ \forall x (F(x) \supset G(x)) \]

where

\[ F(x): \text{x is a METRIC ACCENT} \]
\[ G(x): \text{x is a PHYSICAL OBJECT} \]
\[ H(x, y): \text{x is a property of y} \]

\[ \forall x (F(x) \supset G(x)):\]
A METRIC ACCENT IS A PHYSICAL OBJECT

\[ \forall x (\exists y F(y) \land H(x, y)) \supset \exists y (G(y) \land H(x, y)):\]
A PROPERTY OF A METRIC ACCENT IS THAT OF A PHYSICAL OBJECT,

then, a proof of this inference can be carried out, i.e., the conclusion of this inference can be derived, in the following way without considering what predicates “F,” “G,” and “H” are:

\[
\begin{align*}
(1) & \quad \forall x (F(x) \supset G(x)) & \text{P} \\
(2) & \quad \exists y (F(y) \land H(u, y)) & \text{P} \\
(1) & \quad F(f) \land H(u, f) & \text{EI} 2 \\
(1, 2) & \quad G(f) \land H(u, f) & \text{UI} 1 \\
(1, 2) & \quad \exists y (G(y) \land H(u, y)) & \text{TF} 3, 4 \\
(1, 2) & \quad \exists y (F(y) \land H(u, y)) \supset \exists y (G(y) \land H(u, y)) & \text{C} 2, 6 \\
(1, 2) & \quad \forall x (\exists y F(y) \land H(x, y)) \supset \exists y (G(y) \land H(x, y)) & \text{UG} 7 \\
\end{align*}
\]

Example 9. Formal Derivation of a Belief about a Metric Accent

In this way, logicians and mathematicians such as De Morgan, George Boole, and Ernst Schröder in the late 19th century reformulated and transformed intensional classical logic into extensional logic. As a result, for logicians, neither the process of the formation of extension \( \{x | F(x)\} \), or a set, nor the role of \( F(x) \) matters any more.

For music theorists, however, the process of the formation and the roles of a class of musical entities—that is, finding and identifying currently unknown musical kinds such as “analytical objects” in an atonal composition—are always of primary interest. Since some musical kinds are intentionally chosen or created by composers, it also seems quite natural for music theorists to talk about their intentions or purposes. Music theorists often discuss, for example, for what purpose or why a metric ambiguity at a particular place in a piece is created, why a composer chooses only some particular pc-sets or tone rows, and so on. Such statements expressing intention can hardly be represented in formal terms, however. On the contrary, in modern science, which is in part characterized by the extensive use of mathematics, intentional propositions such as “For what purpose or why was the universe created?” are deliberately excluded so that mathematical languages, or formal logic, are introduced. This is a reason why mathematics plays only limited roles in music theory.

Of course, we can answer questions such as “Why did Stravinsky choose only those tone-rows?” as long as we know the principles, or atonal kinds and associated rules, that govern his compositional choices. In that case, the answer can be derived from the rules and we can acquire new beliefs, as Example 9 shows, in a formal way.

Mathematics or formal logic is, however, not useful for answering a question like this.

It is only after musical kinds and their determinant properties are identified that a formal system consisting of “sets” can be useful for deriving new beliefs or for measuring similarities and
PROXIMITIES. In order to represent not “strange creatures” but musical kinds in set-theoretical terms, we need to know in advance what musical entities are musical kinds and what properties are the “certain characteristics” (Goodman 1972, 440), otherwise pc-set theory and transformational theory will remain for the most part powerful representation schemes that facilitate notational economy and taxonomies.

METAPHOR AND KNOWLEDGE ACQUISITION

Now, our task is to find how we identify currently unknown musical kinds, that is, how we acquire new beliefs about musical kinds that cannot be derived from already known beliefs by set-theoretical operations. The following excerpt from Krumhansl 1990 shows a good example of knowledge acquisition.41

Krumhansl (1990, 18) argues that:

Considerable empirical work supports the general notion that human cognitive and perceptual systems invest certain elements with special status: these elements are given priority in processing, are most stable in memory, and important for linguistic descriptions. This description of a hierarchical ordering of category members would seem readily applicable to tones in tonal contexts. A tonal context designates one particular tone as most central.

Here she hypothesizes that a certain specific case is a member of a well-known class. What I shall try to do in the following is to find how she has acquired this hypothesis, or a new belief. This argument comprises in essence two statements, S1 and S2:

S1: Considerable empirical work . . . for linguistic descriptions.
S2: This description . . . applicable to tones in tonal contexts.

Each statement and the entire argument can be summarized as follows:

| S1a: A process is operative in cognitive systems. |
| S2: The process is also operative in the system for music. |

This is a type of reasoning called analogical reasoning, by which some processes, relations, or properties normally associated with the antecedent are mapped as a model to the domain of the consequent.

Note that, in this argument, two different domains, namely, the cognitive system for music and other systems are associated with each other. This is exactly what metaphor also does. Physical metaphors such as AN EVENT AT A TIMEPOINT IS STRONG and A TONIC IS STABLE are instantiations of the following analogical reasoning:

A principle is operative in the physical domain.
The principle is also operative in the music domain.

Kramer’s discussion examined earlier also relies on this reasoning. In his argument, the balance principle is transferred, or mapped, to the music domain because of this reasoning. In other words, metaphorical transfer is essentially equivalent to non-deductive, analogical reasoning.

Now, let us examine how Krumhansl’s analogical reasoning is carried out. S1 maintains that the general notion “cognitive systems have a stable element,” which is a universal rule, is obtained from a number of instances by the following inductive generalization:

Cognitive system A has a stable element.
Cognitive system B has a stable element.
Cognitive system C has a stable element.

S1a: Cognitive systems have a stable element.
Here, “Considerable empirical work” provides the universal rule with inductive support. In other words, the production of the new metaphor, or the acquisition of the new belief, SOME MUSICAL ENTITIES ARE STABLE is made possible through inductive generalization. Therefore, the production of a new metaphor results from a new hypothesis, and hence conventional metaphors are instances of accepted hypotheses. It is by way of the inductive generalization that Krumhansl has found some similarity between the cognitive system for music and those of others. Naïve set theory fails to represent the formation of a metaphor, a kind, and a similarity because the formation is essentially a process of inductive generalization, for which no definitive formal model has been and perhaps will never be formulated.

Finally, the consequence of her argument is deductively derived as follows:

S1a: Cognitive systems have a stable element.
There is a cognitive system for music.

S2: The cognitive system for music also has a stable element.

The soundness of this deduction is determined by the strength of the inductive inference that derives S1a. Thus, the appropriateness of a metaphoric transfer ultimately depends on the degree of inductive strength.

In this way, the acquisition of a new belief about a musical kind that is not in a known, coherent system and hence cannot be derived by deduction most likely results from the production of a new analogy or metaphor. This is because, as Lakoff and Johnson (1980, 25, 59, 115) point out, we conceptualize less delineated entities such as those found in music in association with other experiences that we understand in clearer terms such as those of spatial orientation and physical objects. In this regard, Goodman (1972) argues that:

The fact that a term applies, literally or metaphorically, to certain objects may itself constitute rather than arise from a particular similarity among those objects. Metaphorical use may serve to explain the similarity better than—or at least as well as—the similarity explains the metaphor. ... I suspect that rather than similarity providing any guidelines for inductive practice, inductive practice may provide the basis for some canons of similarity. (440–41; emphasis added)

Consequently, the identification of currently unknown musical kinds characteristic of atonal music is accomplished, at least at its initial stage, not in a formal but in an experience-based, intuitive manner. It seems quite likely that an atonal theory that is not a collection of rules of representation or taxonomies but designates and classifies atonal kinds comes with a coherent collection of metaphorical expressions.

METAPHORS THAT DESIGNATE ATONAL KINDS

Now, we first need to do “inductive practice” to identify atonal kinds and then proceed to find what properties are the “certain characteristics.” The time-related metaphors coined by Kramer (1988) are currently the only classifiers available for designating similarities, or differences, in atonal music. He maintains that “Most twentieth-century pieces exhibit ... characteristics of several different temporalities” (1988, 58, 61) and designates them by metaphors such as GOAL-DIRECTED LINEAR TIME, NONDIRECTED LINEAR TIME, MULTIPLY-DIRECTED LINEAR TIME, MOMENT TIME, and VERTICAL TIME, which are generalizations of his auditory experiences. So, a hypothesis to be examined here should be the following: Different kinds of atonal passages are characterized by what Kramer calls “different temporalities” and the differences at least partly result from different ways of pitch organization. Then, what we need here is some means that enables us to associate the different temporalities with different ways of pitch organization.

Such an association is achieved, of course, by metaphor and it seems that POSITION-FINDING (Browne 1981) and the “Intervallic Rivalry Model” (Butler 1989, 1992) make the association possible. Browne argues that, because of the unique multiplicity property of the
interval vector of the diatonic set, when we hear tonal music, we constantly try to find our PO-
SION in a particular diatonic set with the help of rare interval classes such as 6 and 1. Butler
(1989) elaborates this idea and argues that:

\[ \ldots \text{Any tone will suffice as a perceptual anchor—a tonal center—until a better candi-
}
\[ \ldots \text{date defeats it. The listener makes the perceptual choice of most-plausible tonic on}
\[ \ldots \text{the basis of style-bound conventions in the time ordering of intervals that occur only}
\[ \ldots \text{rarely in the diatonic set; that is, minor seconds (or enharmonics) and the tritone.}
\[ (238)

In addition, according to Butler and Brown (1981, 1984), we need as few as three pitch-classes,
two pitches a tritone apart and another single tone as “a reliable aural cue to tonic” (53) to carry
out tonic identification judgments.

Although Butler’s model is that of key-finding, it seems that POSITION FINDING does not
have to be restricted to tonal contexts. The arguments in the rest of this paper presuppose the
following hypothesis: If the unique multiplicity property of the diatonic set is the sufficient condi-
tion for POSITION FINDING in the diatonic field, it should be also operative in modal contexts,
where a FOUND POSITION is related not necessarily to a tonic but to a particular unordered
diatonic set.\textsuperscript{47} If this is the case, Butler’s model has significant implications for atonal music as
well. Dubiel (1991) suggests that:

\[ \ldots \text{a pitch-class-set analysis of any reasonably complex tonal piece \ldots would be bound}
\[ \ldots \text{to involve a distinction between the diatonic collection as presented and the diatonic}
\[ \ldots \text{collection as referred to. The possibility of making such a distinction in non-diatonic}
\[ \ldots \text{contexts should be kept in mind. (1–2)}

In other words, it seems possible that we still carry out POSITION-FINDING in some atonal
music in which a diatonic set is not present in its entirety but can be referred to with the aid
of rare interval classes. Browne’s metaphor and Butler’s model may well be extended to explain
how we perceive some kinds of atonal music.

If the hypothesis is correct, there is a good reason why the first classifiers of atonal music are
time-related. According to Lakoff and Johnson (1980), the notion of time in English is structured
around the general metaphor TIME IS A BOUNDED SPACE WHERE WE MOVE. Therefore,
the metaphoric expression WE MOVE IN A MUSICAL SPACE is derived as follows:

\[
\text{A TEMPORAL ENTITY IS A BOUNDED SPACE WHERE WE MOVE.}
\]
\[
\text{SOME MUSICAL ENTITIES ARE BOUNDED SPACES}
\]
\[
\text{WHERE WE MOVE, or WE MOVE IN A MUSICAL SPACE.}
\]

Since a position is found in a certain space, Browne’s metaphor POSITION FINDING implicitly
presupposes this reasoning.\textsuperscript{48}

The association between musical entities and their spatialization on a staff sheet seems present
in Kramer’s theory of musical time, that is, horizontal staves are CONTAINERS associated with
time. It is for this reason that Kramer characterizes the music that has no MOTION, or no
DIRECTEDNESS, as STASIS or VERTICAL TIME, IN WHICH MUSICAL ENTITIES MOVE
not IN HORIZONTAL but VERTICAL DIMENSION. “Variation 2, 4, and 10” from Stravinsky’s
\textit{Variations} may be, as Kramer (1988, 210–13) suggests, good examples of the music of this kind.

Although detailed analyses are beyond the scope of the present paper, some examples help us
understand the relevance of POSITION-FINDING to atonal pitch organization. Butler’s model
seems to explain, for instance, the GOAL-DIRECTEDNESS in the Schoenberg excerpt (Exam-
ple 5). The reason why m. 3 sounds like $D$ minor may be that, as Example 10 shows, since NO
POSITION IS FOUND up to m. 2 but in m. 3 there are two semitones, $\{B^\flat, A\}$ and $\{E, F\}$, and
a single tritone, $\{B^\flat, E\}$, all of which are contained in a diatonic set, we FIND OUR POSITION,
or GOAL, in C scale on F with D♯, or C♯, as the leading tone in D minor.

Example 10. POSITION-FINDING in Schoenberg’s op.11, no.1

Van Egmond and Butler (1997, 19–20) also point out that \{0,1,4,7\}, the \(T_n\) type of the chord \(\{B^\flat 2, A3, D^\flat 4, E4\}\) in m. 3, occurs only once exclusively in a single harmonic minor key, in this case, D harmonic minor.

Obviously, there are the two most distinct states of POSITION-FINDING in an atonal passage, that is, a POSITION is either FOUND or LOST. Therefore, in addition to those proposed by Forte (1973) and Hasty (1981b, 1984), there seems to be another possible guideline for segmentation. That is, on the crudest level, since POSITION FINDING is controlled especially by tritones, segmentation can be carried out so that each pc-set, or an “analytical object,” in a piece is either a subset of a diatonic collection with a single tritone or else it is not. We may be able to consider these two types of pc-sets as most distinct atonal kinds.

Now, following this hypothetical guideline, let us examine a rather extreme case, that is, again, Webern’s Variationen op.27, which may require the most dense POSITION-FINDING.

Measure: 1 1 2 2
Motive: \(a_1\) \(a_2\) \(b_1\) \(b_2\)
PC-Set: 3-5 3-5 3-5 3-8

Example 11. Transitions of POSITIONS in Variationen

Example 11 shows that a single tritone is contained in each consecutive trichord, which is a subset of a diatonic collection. Each of the two complementary motives, \(a\) and \(b\), consists of the two consecutive trichords, \(a_1\) and \(a_2\) and \(b_1\) and \(b_2\). In motive \(a\), the POSITION SHIFTS two steps clockwise between \(a_1\) and \(a_2\), while in \(b\) it shifts one step counter-clockwise between \(b_1\) and \(b_2\). It follows that the opposite direction of the transitions of POSITIONS corresponds to the inverted relation of the melodic contours between \(a\) and \(b\) (see Example 4). While these SHIFTS are rather closely related transpositions by \(T_2\) and \(T_5\), there is a LEAP of POSITIONS between \(a\) and \(b\), namely, \(T_3\), that marks the boundary between the two motives. In short, the states of POSITIONS are closely coordinated with the motivic formation, rhythms, and contours.
Those implied diatonic collections show that there are different states and transitions of POSITIONS. That is, a POSITION may be in a FOUND or LOST state, in transition from FOUND to LOST, and vice versa, and as part of SHIFTS or LEAPS. Kramer’s “MULTIPLY-DIRECTED TIME” may result from the mixture of SHIFTS and LEAPS. As mentioned earlier, “Variation 2, 4, and 10” from Stravinsky’s Variations exhibit STASIS or VERTICAL TIME because a reference to a diatonic collection is impossible, that is, we have NO POSITION TO FIND. In this way, some atonal music such as Stravinsky’s Variations and Schoenberg’s op.11/1 juxtaposes DIRECTED sections or passages with STASIS perhaps because the juxtaposition provides ways of POSITION-FINDING with the maximum variety.

In tonal music, we always try to find our POSITIONS and DIRECTIONS with respect to the tonic and the diatonic collection in the main key, which is the absolute referential point, toward which DIRECTED MOTIONS can be activated and from which DISTANCES are measured. Also, as Browne (1981) points out, because of the unique multiplicity property, the various transpositions are hierarchically related to the referential set in the tonic key by their various common-tone distributions. By contrast, the observation so far seems to suggest that, in some kinds of atonal music, implied diatonic collections are, of course, not related to a single referential diatonic collection. Instead, it seems that those implied diatonic collections adjacent in a time dimension are related to each other in terms of their relative distances on the circle of fifths. In some atonal contexts, pitches are organized in such a way that we can FIND POSITIONS, and the differences in the relative distances and transitions among implied diatonic collections may result in “different temporalities.”

The hypothetical guideline of segmentation suggests a classification of pc-sets as well. That is, the classification of all the trichords, tetrachords, and pentachords into the following three Tritone-Sensitive Collections, TS-Collections for short, may be useful:

- TS-Collection 1: the subsets of the diatonic set without a tritone
- TS-Collection 2: the subsets of the diatonic set with a tritone
- TS-Collection 3: the rest of the pc-sets

With a pc-set from TS-Collection 1, if a POSITION is already FOUND, what Browne (1981) calls “pattern-matching” happens and the POSITION IS CONFIRMED, if not, POSITIONS ARE SUGGESTED; a pc-set from TS-Collection 2 helps us find our POSITION in a particular diatonic collection; and with one from TS-Collection 3, a POSITION IS LOST.

Needless to say, some pitches in a piece are more salient than others due to their registers, dispositions in a chord, dynamics, durations, and so on. In addition, Butler and Brown (1981) and Brown, Butler, and Jones (1994) point out that temporal orders of pitch classes affect POSITION FINDING. Thus, we need to further calibrate the degrees of implications for POSITIONS by adding more criteria. Since the relationships among those factors are so complex, however, we cannot help but start with these crudest criteria. Van Egmond and Butler (1997, 17–23) suggest that there are some \( T_n \) types size 3 to 6 which occur exclusively in any of the three types of diatonic collections, namely, those of major or pure minor, harmonic minor, and ascending melodic minor. Although they are POSITION-determiners in tonal contexts, the classification should be taken into account when we try to refine the criteria.

Next, let us examine whether or not the classification above helps explain why Stravinsky chose only some particular twelve-tone rows. If POSITION FINDING is relevant to how we perceive atonal music, it must govern composers’ choice of pitch materials as well. Because of the different levels of combinatoriality, some Row-Class members do not generate all three TS-Collections of their subsets. Only those subsets checked in Table 4 are available for each Row Class:
Table 4. Availability of TS-Collections in Row Class

<table>
<thead>
<tr>
<th>Row Class</th>
<th>Trichords</th>
<th>Tetrachords</th>
<th>Pentachords</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2-3</td>
<td>✓  ✓  ✓</td>
<td>✓  ✓  ✓</td>
<td>✓  ✓  ✓</td>
</tr>
<tr>
<td>1-2-3</td>
<td>✓  ✓  ✓</td>
<td>✓  ✓  ✓</td>
<td>✓  ✓  ✓</td>
</tr>
<tr>
<td>1-2-3</td>
<td>✓  ✓  ✓</td>
<td>✓  ✓  ✓</td>
<td>✓  ✓  ✓</td>
</tr>
<tr>
<td>1-2-3</td>
<td>✓  ✓  ✓</td>
<td>✓  ✓  ✓</td>
<td>✓  ✓  ✓</td>
</tr>
<tr>
<td>1-2-3</td>
<td>✓  ✓  ✓</td>
<td>✓  ✓  ✓</td>
<td>✓  ✓  ✓</td>
</tr>
<tr>
<td>1-2-3</td>
<td>✓  ✓  ✓</td>
<td>✓  ✓  ✓</td>
<td>✓  ✓  ✓</td>
</tr>
<tr>
<td>1-2-3</td>
<td>✓  ✓  ✓</td>
<td>✓  ✓  ✓</td>
<td>✓  ✓  ✓</td>
</tr>
<tr>
<td>1-2-3</td>
<td>✓  ✓  ✓</td>
<td>✓  ✓  ✓</td>
<td>✓  ✓  ✓</td>
</tr>
</tbody>
</table>

Stravinsky chose tone rows only from Row Classes 1, 2, and 7. Table 4 shows that only those hexachords of Row Classes 1, 2, and 3 can generate trichordal, tetrachordal, and pentachordal subsets of all the three TS-Collections. In contrast, the tone rows of Row Classes from 4 to 9 do not generate subsets of all the three TS-Collections. The tone rows of Row Classes 4, 6, and 8 in particular generate none of the subsets of TS-Collection 2. Meanwhile, the tone rows from Row Classes 5, 7, and 9 generate trichordal subsets of all the three TS-Collections. The hexachords of Row Class 9, however, generate only three distinct trichords, namely, 3-6, 3-8, and 3-12. In addition, those trichords do not have interval-classes 1, 3, and 5, which permeate Stravinsky’s music. Thus, it may be even a matter of course for Stravinsky to have chosen no tone rows from Row Classes 4, 6, 8, or 9. Five distinct trichords are derived from Row Class 7 and seven from Row Class 5. A tone row from Row Class 7 is chosen for Movements. Despite its extremely low relative frequency, Row Class 7 shares the same property with Row Classes 1, 2, and 3, that is, it is capable of generating subsets of all three TS-Collections and hence all states and transitions of POSITIONS, or “different temporalities,” can take place. It may not be an accident that in Movements, because its tone row generates only trichordal subsets of all three TS-Collections, the texture of those sections in which series forms are used as melodies is rather sparse. In fact, the sustained tetra- and pentachords are derived from verticals of a rotational array of the tone row or “arbitrarily” chosen like those in Canticum Sacrum as described below.

The choice and the usage of the tone rows of Stravinsky’s earlier twelve-tone pieces Canticum Sacrum and Agon also seem to illustrate his intention to use pc-sets from all the TS-Collections. He chose for these pieces tone rows from Row Class 6, which do not generate all three TS-Collections. In these two earlier twelve-tone pieces as well as Threni, for which a tone row from Row Class 1 is chosen, the tone rows are not treated as pairs of unordered hexachords. For example, in Canticum Sacrum, the chords played by the trombones in mm. 293-94, \{5, A, B\}, and by the organ in mm. 304-6, \{1, 2, 3, 7\}, are “arbitrarily” derived from the tone row in the following way:

<table>
<thead>
<tr>
<th>Measures: 293–94</th>
<th>Series form: (&lt;4,6,9,5,8,7,0,A,B,1,3,2&gt;)</th>
<th>Chord: {5, A, B}</th>
<th>Forte name: 3-5 (TS-Collection 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measures: 304-6</td>
<td>Series form: (&lt;7,6,8,A,B,9,2,1,4,0,3,5&gt;)</td>
<td>Chord: {7, -2,1, -3}</td>
<td>Forte name: 4-5 (TS-Collection 3)</td>
</tr>
</tbody>
</table>

Example 12. “Arbitrary” Choice of PC-Sets in Canticum Sacrum

Neither 3-5 nor 4-5 can be derived from the hexachords of the tone row. Therefore, perhaps because Stravinsky intended to use pc-sets from all three TS-Collections, he chose no tone rows from Row Class 6 after starting to apply hexachordal division.

METAPHOR AND THEORY CHANGE

Transfers of properties in a metaphor seem, as Way (1991, 145–46) argues, open-ended and indeterminate. This may be a crucial difference between a metaphor and a literal expression or a simile. In the case of simile, since “everything is like everything” (Davidson 1984b, 254), because of the word “like”—which implies the existence of at least one property shared by the source and the target—no conflict between the two subjects would occur. In other words, the
degree of prominence of a transferred property in a metaphor could always be lower. It is this open-endedness of metaphoric transfer that plays another major role of metaphor in music theory. Since transferred and highlighted properties are to some extent related and predictable due to the order in the source, some metaphors serve as heuristics and determine a direction of the development of a theory in which such metaphors are employed. \(^{50}\)

Recalling the earlier discussion regarding the controversy over metric accents, the disagreement between Kramer and Cone may stem from the open-ended and indeterminate nature of metaphoric transfer. That is, when:

\[
\text{Metric Accent} = \{x | a_1(x) \land a_2(x) \land \cdots \}
\]
\[
\text{Physical Object} = \{x | p_1(x) \land p_2(x) \land \cdots \}
\]

and

\[
p_1(x): \text{"x exerts strong/weak force"}
\]
\[
p_2(x): \text{"x is subject to the balance principle,"}
\]

while, in Kramer’s metaphor, metaphoric transfer proceeds up to \(p_2\) in \((P, \preceq)\):

Kramer’s \(m_{\text{Metric Accent}} = \{x | p_1(x) \land p_2(x) \land a_1(x) \land \cdots \}\),

it remains only for \(p_1\) in Cone’s:

Cone’s \(m_{\text{Metric Accent}} = \{x | p_1(x) \land a_1(x) \land \cdots \}\).

It could be said that the process of Kramer’s theory formation is guided by prominent properties in the source domain of the general metaphor. In this sense, the partial order of prominent properties in the source domain has served as a heuristic. In other words, the course of the development of Kramer’s theory of rhythm and meter may have been predetermined by his adoption of the physical metaphors.

As discussed earlier, in order for an entire collection of classes to maintain deductive consistency, transferred properties of the members of a class must be inherited in all its subclasses. Therefore, a theory develops in such a way that transferred properties are inherited from a superset to its subsets. In addition to the process of possibly increasing instances of metaphoric transfer in accordance with a partial order of properties, this inheritance of properties seems to be another factor of the development of a theory.

In general, as Black (1993, 30) observes:

Every implication-complex supported by a metaphor’s secondary subject [source] . . . is a model of the ascriptions imputed to the primary subject [target]: Every metaphor is the tip of a submerged model.

Accordingly, Carroll and Mack (1985, 48–49) observe that:

It seems to us inherent in the nature of metaphor that its relation to a metaphorized object of domain be not just incomplete, but indeterminate. . . . It is this property of metaphor that affords cognitively constructive processes which can lead to new knowledge.

The changes in the use of physical metaphors in the theory of rhythm and meter can be traced with relative ease. In the infancy of the theory, Cooper and Meyer (1960) did not use the coupled terms designating opposite PHYSICAL FORCE, namely, STRONG and WEAK. In other words, they still did not fully adopt physical metaphor. Since then, Cone and other theorists working on rhythm and meter have been developing physical metaphors, which Kramer’s theory epitomizes. We do not know, however, how far the transfer of properties extends because, as discussed, it is contingent on inductive generalization. The more it extends, the more the entire theory becomes at least coherent, although not all properties can be transferred.
“Compared to tonal theory, now in its fourth century of development, post-tonal theory is in its infancy. As a result, there are still substantial areas of disagreements” (Straus 2000, vii). Since atonal theory is still in its infancy, it needs some experience-based metaphors to designate and classify unknown musical kinds. The collection of metaphorical terms so introduced in atonal theory hence identify similarities among their members, but may also cause disagreements just as the physical metaphors did between Cone and Kramer. Then, “...the similarity notion, starting in its intuitive phase, developing over the years in the light of accumulated experience, passing then from the intuitive phase into theoretical similarity...” (Quine 1969, 136ff), as the theory evolves, may be redefined and written in more theoretical or set-theoretical terms. As a theory develops, experience-based notions will be replaced by more theoretically entrenched ones. Time- and space-related metaphors may be replaced by more theoretical notions as the theory develops and formal systems such as that of set complexes (Forte 1964, 1965, 1973) and various taxonomies of pe-sets and transformations will provide a powerful framework for reasoning on atonal kinds and the formation of atonal theory.

To overview what we have discussed here, since music theorists deal with ever changing intentions of composers and theorists, the rules of style change and theory change should govern the most fundamental constraints, on which any compositional decisions and choices of “analytical objects” necessarily depend. Thus theories of theory change and style change are foundations for not only atonal theory but also any theories of music.

NOTES

1Earlier versions of this paper were presented as “On the Relevance of Metaphor to Music Cognition” at the Society for Music Perception and Cognition Conference, University of California at Berkeley, Berkeley, California, June, 1995, and “An Extension of Richmond Browne’s POSITION-FINDING to Atonal Contexts” at the Fourth International Conference on Music Perception and Cognition, McGill University, Montreal, Canada, August, 1996.

2The term “atonal kinds” is named after “natural kinds” (Quine 1969). I shall discuss Quine’s view and differences between “kind” and “set” or “class” later. Although DeBellis (1995) and Zbikowski (2002) among others use “category” instead of “kind,” I prefer the latter because of the context in which Quine makes clear the distinction between “set” and “kind.”

3Following the convention adopted by Lakoff and Johnson (1980), metaphorical expressions are henceforth written in capital letters.

4See Mead 1989 for an overview of the state of research in atonal music. Although its bibliography is no longer up to date, his assessment seems still plausible today.

5Meyer (1989, 340) argues that “The melodic relationships employed by most twentieth-century composers tended to be those least dependent on tonal syntax. Similarity relationships—varied sequence, partial imitation, complementary structures—were the most important means for creating process and coherenc” For more examples from Webern, see Hasty 1988.

6Those who wonder if the sense of the GOAL-DIRECTED MOTION towards D4 in m. 4 might be created because of the DESCENDING LINEAR PROGRESSION from B4 in m. 1 to E4 in m. 3 with A4 in m. 2 as a gap-filling are encouraged to play just mm. 3-4 to find that the PROGRESSION is not exclusively responsible for the MOTION.

7As a rare case, tetrachordal division is employed in Introitus. I owe this observation to Prof. Joseph Strauss.

8The number of distinct series forms generated by a tone row from Row Class 9, for example, is calculated in the following way: Let <Hn, Hb> be an ordered pair of complementary unordered hexachords Ha and Hb. Since the tone row is fourth-order all-combinatorial at six different pitch levels,

\[
RT_n I <Hn, Hb> = T_{n-1} <Hn, Hb>,
RT_n <Hn, Hb> = T_{n+1} <Hn, Hb>,
T_n I <Hn, Hb> = T_{n-1} <Hn, Hb>,
\]

where \( n = 1, 2, 3, ... , 11 \), hold for some \( i \); that is, RI, R, and I series forms duplicate P series forms. In addition, \( T_n <Hn, Hb> = T_{n+i} <Hn, Hb>, \) for \( i = 0, 2, 4, 6, 8, 10, \) holds; that is, there are only two distinct transpositional levels. Therefore, because the number of permutation of a single hexachord is 6!, the ordered pair of hexachords produce 6!/2 = 1,036,800 distinct series forms.

9Hasty (1988) also discusses issues concerning the relations between particular pitch materials such as tone rows and pre-compositional, general principles.

10Morgan 1980 makes another contribution to the earliest literature on metaphor in music theory.

11Lochhead 1989; Rust 1994, and Mead 1999 are rare examples that discuss or employ metaphor in relation to atonal music.

12In logic and philosophy, “truth,” “correctness,” “soundness,” “validity,” and “consistency” have distinctive usages, which are observed in the present paper.


14“Argument” is used here in a standard sense in logic, that is, a derivation of a conclusion from premises.
“Naive set theory” is a theory developed by Georg Cantor. It was later reformulated as “axiomatic set theory” to cope with paradoxes found by Bertrand Russell. For a further discussion about knowledge representation, see Way 1991, Sowa 2000, and Brachman and Levesque 2004.

See Camac and Glucksberg 1984 for a further discussion about the notion of association.

A stress accent can be identified by its amplitude, which is measured in Newtons per square meter \((N/m^2)\), that is, as a “force” applied over an area.


One might think that A STRONG ACCENT should be COUNTERBALANCED by a NEGATIVE STRONG ACCENT as Schema 1 below shows:

Comparing with ball-throwing, Cone (1968, 26–27) supports accentual pattern 1 in Example 6 as follows: “[a rhythmic] principle [on some level] must be based on the highly abstract concept of musical energy... Unlike the undifferentiated transit of the ball, the musical passage is marked by stronger and weaker points. . . .” (emphasis added). In other words, since a ball-throwing is charged a physical force only at its inception and the force neither increases nor decreases during the throwing, temporal gain of force (STRONG) must be compensated by the equal amount of loss (WEAK). Thus, Cone and perhaps Kramer as well seem to support Schema 2. The preference for Schema 2 may also be the reason why Kramer, as discussed later, believes that metric irregularity, or PHYSICAL IMBALANCE, can be resolved on a higher metric level, i.e., WEAK and STRONG HYPER-METRIC ACCENTS counterbalance each other because, otherwise, initially charged MUSICAL ENERGY would either diminish or eventually explode. A preference for one of the schemata depends on a “metaphysical” standpoint as to the nature of MUSICAL FORCE. See Rothfarb 2002 for various modelings of MUSICAL ENERGY.


More elaborate integration of Black’s theory into a formal system is found in Way 1991, to which I owe much for the set-theoretical representation of metaphor. Steinhart 2001 is another excellent attempt at the formalization of reasoning with metaphors.

The distinction between “knowledge” on the one hand and “belief”—as probable knowledge—on the other is a traditional dichotomy in philosophy.

The order here should be “partial” since there may be two properties between which comparability does not hold. A partial order in a set \(X\) is defined as a reflexive, antisymmetric, and transitive binary relation in \(X\). That is, for all \(x, y, \) and \(z \) in \(X\), we have: (i) \(x \leq x\); (ii) if \(x \leq y \) and \(y \leq x\), then \(x = y\); (iii) if \(x \leq y \) and \(y \leq z\), then \(x \leq z\). The inequality sign \(\leq\) is customarily used for the expression of a partial order.

The terms “target” and “source” correspond to “tenor” and “vehicle” introduced by Richards (1936b) and Hesse (1980).

Gentner (1985) proposes the structure-mapping theory of analogy, which is closely related to metaphoric transfer.

We are using here a system of formal logic. Since, as pointed out earlier, a system of formal logic is a model, even if the representation of reasoning on a contradictory statement such as the example below consists of a rather long sequence of steps of manipulations of concepts, the actual process of the same reasoning may take place instantly in our brain.

\[
\begin{align*}
\{x, & s,M(x)\} \cap \{x, s,W(x)\} = \emptyset, \text{ that is, } \neg \exists x (s,M(x) \land s,W(x)), \text{ then,} \\
\neg \exists x (s,M(x) \land s,W(x)) \land \neg x(s,M(x) \land s,W(x)) \\
\neg \exists x (s,M(x) \lor \neg s,W(x)) \\
\forall x s(M(x) \lor \neg s,W(x)) \\
\forall x (s,M(x) \land s,W(x)) \\
\forall x (\neg s,M(x) \lor \neg s,W(x)) \\
\text{Man is not a wolf.}
\end{align*}
\]

Therefore, it is not surprising that the understanding of a contradictory statement takes less processing time than that of a non-contradictory one in psychological experiments (Gibbs 1994, 1997). It is simply because our brain may be wired that way.

When the target is a proper name as in “Juliet is the sun,” a metaphor does not create a subclass.

Black (1962a, 41) metaphorically calls this “reorganization” in the target domain “filtering” and “using language... as a lens for seeing [the target].”

This reasoning pattern is equivalent to a mode of syllogistic inference called the Barbara mode. Needless to say, Aristotle and medieval logicians did not think about syllogism “extensionally” as represented here but “intensionally” in terms of categorical judgment. Advantages and disadvantages of extensional logic will be discussed later.
The following discussion owes much to Quine 1969. Quine’s view, known as “the naturalistic conception of epistemology,” is further discussed by Kripke (1972) and Putnam (1975a).

For a further discussion, see Goodman 1983a, in which he talks about “grue” (green + blue) emerald.

For experimental evaluations of “similarity” as well as other notions employed in pitch-class set theory, see Bruner 1984 and Gibson 1986, 1988, 1993.

In this regard, prototype theory, proposed by Rosch (1973, 1978, 1983) and Rosch and Mervis (1975), may seem useful. As Rosch and Mervis (1975, 573–74) state, however, “[prototype theory] was not intended to provide a processing model of . . . formation of prototypes. . . . For most domains . . . prototypes do not appear to precede the category. . . .” See also Carnap 1967, 141–47 for his principle of the formation of a kind, which is equivalent to the definition of prototypicality (Rosch and Mervis 1975, 602), and criticisms of the principle by Goodman 1951, 163f and Quine 1969, 120–21.

The issues discussed here are only a fraction of them in an enormous research field, where they are relevant to Frege 1892, Russell 1905, Strawson 1950, Quine 1960, Davidson 1984a, Putnam 1975b, Grice 1989, and many other philosophers and recent AI researchers.

This is the same reasoning pattern formulated by Augustus de Morgan to show the limitation of classical logic.

From left to right, the numbers in the first column show the dependency of each statement; the next column show the number of each line; the numbers in the last column show the line-number of a statement used; the rules of inference are the following: P: premise, Q: change in quantifiers, EI: existential instantiation, UL: universal instantiation, TF: truth functional implication, EG: existential generalization, C: conditionalization, UG: universal generalization.

The significance of metaphor in science is discussed by many philosophers. See, for example, Hesse 1966, Kuhn 1993, and Mac Cormac 1976.

The significance of formalization and related issues are discussed in depth by Boretz (1970) and Rahn (1979a, 1979b). Criticisms of them are found in Brown and Dempster 1989, which discusses the nature of scientific music theory. Since the notion of science itself is still a subject of controversy in philosophy of science, the present paper does not examine the criticisms but restricts itself to issues of reasoning.

“Knowledge acquisition” is a generic term and does not necessarily suggest the acquisition of “knowledge” as opposed to “belief” as probable knowledge. This distinction between “knowledge” and “belief” is a traditional dichotomy in philosophy since Plato.

Another instance of reasoning of the same form is an inductive generalization from historical facts, which is also common in music theory. For example, Zbikowski (1997) argues that Meyer’s and Cone’s and Morgan’s analyses of Mozart’s theme diverge because of the two different conceptual models they employ. This argument goes by way of inductive generalization as follows:

The two models were “developed at different points in the history of Western thought” (204) and “have often been used” (200), that is:

The models were used at point A in the history of Western thought.

The models were used at point B in the history of Western thought.

The models were used at point C in the history of Western thought.

... The two models “are in general basic to thought” (195).

Then, “[the two models] are employed in accounts of music” (217) is inferred from this conclusion together with another array of historical facts that the models have been mapped onto the music domain. While “the two models are employed in accounts of music” results from the conclusion because an account of music is an instance of thought, it does not entail “the two models are employed by those three theorists in their accounts of music” unless “the two models are the only ones employed by music theorists” is assumed. Needless to say, however, neither the statements of those historical facts nor the conclusion of the inductive generalization above entails the assumption. At the same time, “their analyses diverge” does not necessarily follow from “they used the two conceptual models” because there are many other possible causes such as the balance principle that, as demonstrated earlier, result in conflicting analyses. In short, “the discrepancies between their analyses result from the two different conceptual models” cannot be confirmed by those historical facts and hence there remains a possibility that the two conceptual models are irrelevant to their analyses. This is an example of a widely known problem of inductive reasoning. See Goodman 1983a, 1983b for the further discussion of the problem and Hempel 1965, 10–20 for the issues of inductive confirmation.

The effects of metaphor and analogical reasoning on music cognition cannot be overemphasized. For instance, Kramer finds hypermeter on higher hierarchical levels whereas some others do not, perhaps because his hearing is somehow guided by the balance principle latent in his physical metaphors. In other words, what we believe that we hear in part depends on the metaphorical expressions we use. Therefore, the experimental result “a tonic is stable,” for example, might exhibit nothing more than the trivial fact that we are trained to metaphorically characterize a tonic as STABLE. Even if we hear a tonic as STABLE, it does not necessarily mean that it is literally “stable” in memory.

Induction with metaphor is also discussed by Cohen and Margalit (1972) and Sternberg, Tourangeau, and Nigro (1993).


Schenkerian theory exhibits remarkable coherence in terms of spatialization and orientation metaphors. Morgan (1980) discusses this aspect of Schenkerian theory. As Strauss (1987) demonstrates, however, it can hardly maintain its coherence if set into a different context such as that of atonal music.
For the distinction between key- and diatonic-set-finding, see Butler 1998 and Brown 1988.

Morgan (1980, 529f) also points out that the notion of musical space is inseparable from that of musical time.

Van Egmond and Butler (1997) classify $T_n$ types with respect to the degrees of implications of tonal centers as well as major and minor modes.

See Mac Cormac 1985 and Boyd 1993 for the role of metaphor in theory change. Since the 80s there has been an increasing attention to the formal modeling of theory change, the basic literature on which includes Makinson 1985, Alchourrón, Gärdenfors, and Makinson 1986, Gärdenfors 1988, Levi 1991, and Fuhrmann and Morena 1991.

The notion of “entrenchment” in relation to the validity of induction is extensively discussed in Goodman 1983a, 1983b.

Meyer 1989 is currently the most comprehensive study in style change.

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